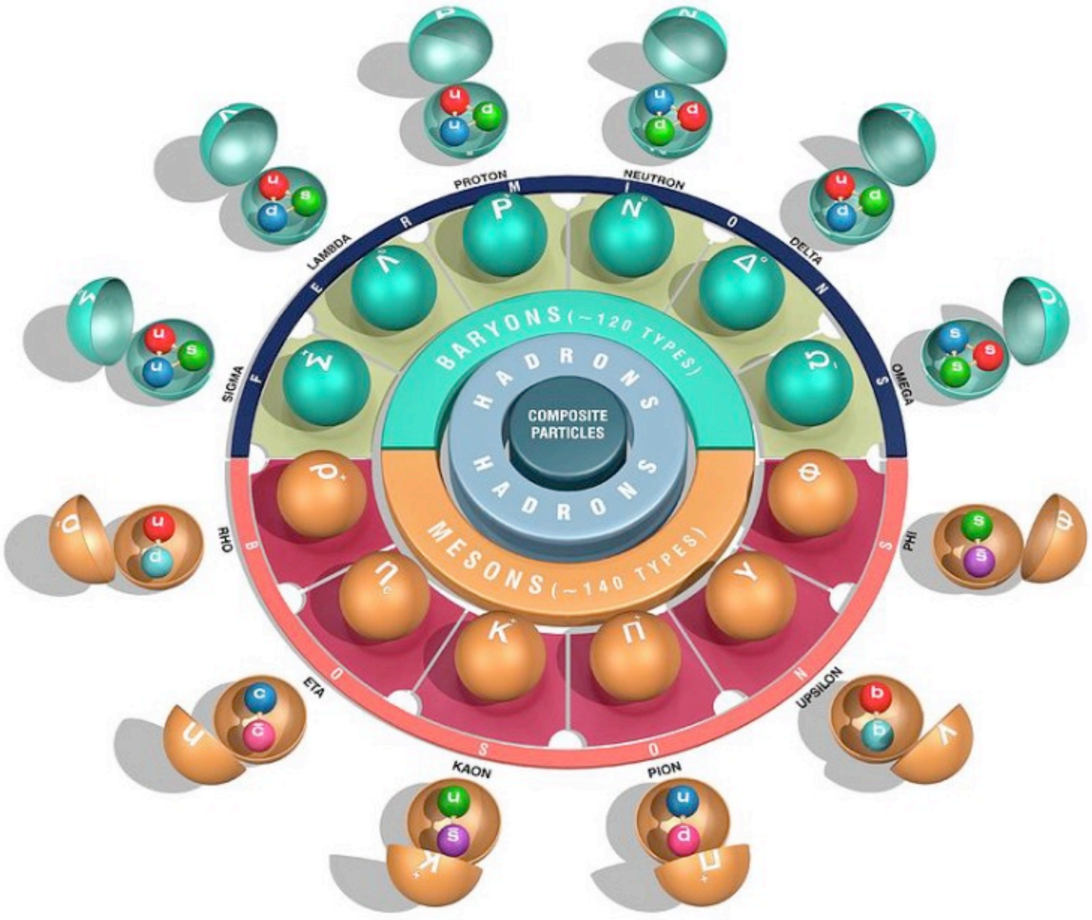


Particle Physics I

Lecture 12: Symmetries and the Quark Model II

Prof. Radoslav Marchevski

November 26th 2024

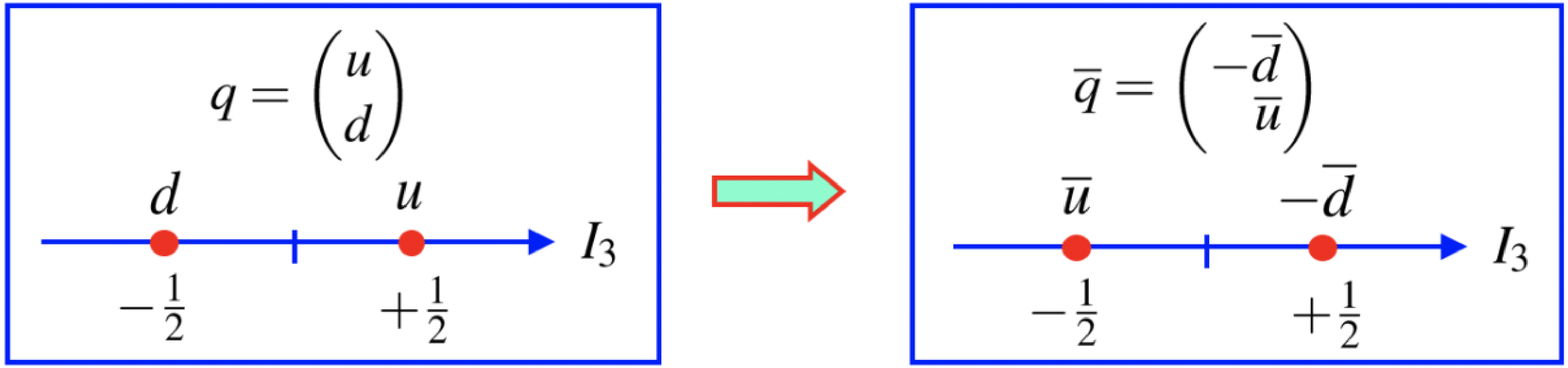
Learning targets

<u>Learning targets</u>

- Introducing $SU(3)$ symmetry: adding an s quark to the mix
- Generators of $SU(3)$ symmetry and link to $SU(2)$: ladder operators
- How to combine quarks (u, d, s) and antiquarks $(\bar{u}, \bar{d}, \bar{s})$ into mesons and baryons
- Classification of hadrons based on their quantum numbers and link to experiment

Recap: isospin, quarks, antiquarks, mesons, and baryons

- The u, d quarks and $\bar{u}, \bar{d}$ antiquarks are represented as isospin doublets



$$\bar{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\bar{d} = -\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- *Note the " – " sign:* the ordering and minus sign in the antiquark doublet ensures that antiquarks and quarks transform in the same way ($q' = Uq$, $\bar{q}' = U\bar{q}$)
  - important if we want physical predictions to be invariant under $u \leftrightarrow d$ and $\bar{u} \leftrightarrow \bar{d}$
- Two types of hadrons: baryons (qqq) and mesons ($q\bar{q}$)

Antiquarks and mesons ($\boldsymbol{u}$ and $\boldsymbol{d}$)

- Consider the effect of ladder operators on the antiquark isospin states:

$$T_+\bar{u} = T_+\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} = -\bar{d}$$

- The effect of the ladder operators on antiparticle isospin states are

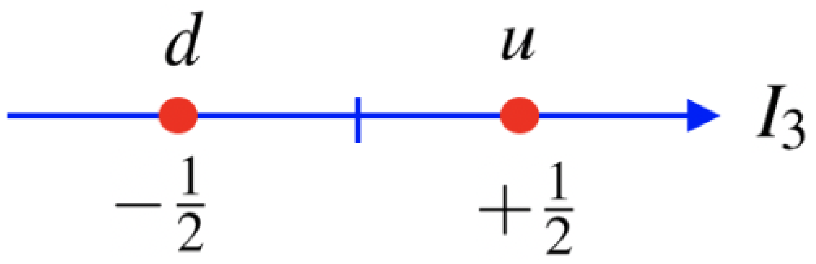
$$T_+\bar{u} = -\bar{d}, \qquad T_+\bar{d} = 0, \qquad T_-\bar{u} = 0, \qquad T_-\bar{d} = -\bar{u}$$

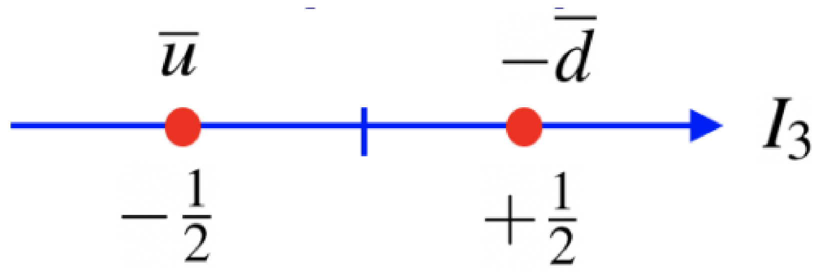
- To be compared with the same operation on quarks

$$T_+u = 0, \qquad T_+d = u, \qquad T_-u = d, \qquad T_-d = 0$$

Light ud mesons

- We can now construct meson states from combinations of up and down quarks





- Consider the $q\bar{q}$ combinations in terms of isospin

$$|1,+1\rangle = \left|\frac{1}{2},+\frac{1}{2}\right\rangle \overline{\left|\frac{1}{2},+\frac{1}{2}\right\rangle} = -u\bar{d}$$

$$|1,-1\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle \overline{\left|\frac{1}{2},-\frac{1}{2}\right\rangle} = d\bar{u}$$

- Where the bar in $\overline{\left|\frac{1}{2},+\frac{1}{2}\right\rangle}$ indicates this is the isospin representation of an antiquark

Light $\boldsymbol{ud}$ mesons

- To obtain the $I_3 = 0$ states we use the ladder operators and orthogonality

$$T_-|1,+1\rangle = T_-[-u\bar{d}]$$

$$\sqrt{2}|1,0\rangle = -d\bar{d} + u\bar{u}$$

$$\Longrightarrow \quad |1,0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

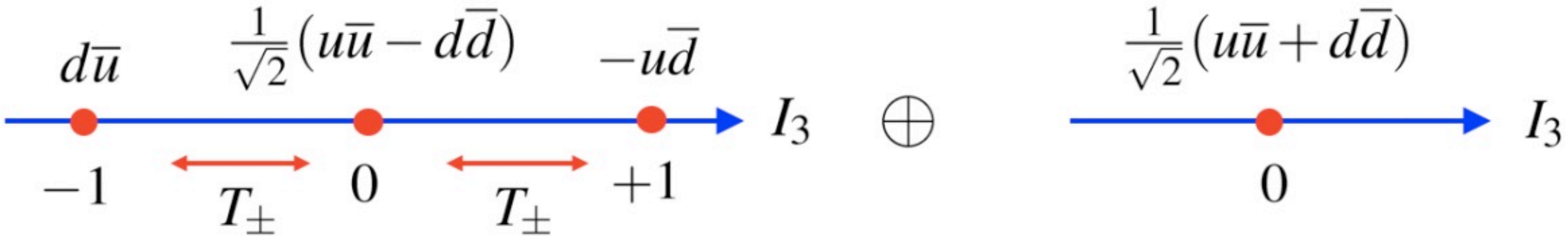
- Orthogonality gives

$$|0,0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

Light $\boldsymbol{ud}$ mesons

- To summarize

- Gives a **triplet** of $I = 1$ states and a singlet of $I = 0$ state



- Usually written as $2 \otimes \bar{2} = 3 \oplus 1$
    - 2 stands for a quark doublet
    - $\bar{2}$ stands for an antiquark doublet

Light $\boldsymbol{ud}$ mesons

- To show that the state obtained from orthogonality with $|1,0\rangle$ is a singlet we can use ladder operators

$$T_+|0,0\rangle = T_+\frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) = \frac{1}{\sqrt{2}}\left(-u\bar{d} + u\bar{d}\right) = 0$$

- Similarly

$$T_-|0,0\rangle = 0$$

- A singlet state is a "dead-end" from the point of view of ladder operators

$SU(3)$ flavour

- Extend these ideas to include the strange quark
  - since $m_s(95 \text{ MeV}) > m_u, m_d(2.2{,}4.7 \text{ MeV})$ there is **no exact symmetry**
  - but m_s is not very different from m_u, m_d and we can treat the strong interaction and resulting hadron states as if it were symmetric under $u \leftrightarrow d \leftrightarrow s$
  - *note:* any results obtained from this assumption are only **approximate** as the symmetry is not exact
- The assumed *uds* flavour symmetry can be expressed as

$$\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = \hat{U} \begin{pmatrix} u \\ d \\ s \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$\boldsymbol{SU(3)}$ flavour

- The 3×3 unitary matrix depends on 9 complex numbers (18 real parameters)
  - there are 9 constraints from $\hat{U}^{\dagger}U = 1$
  - $\Rightarrow$ we can form $18 - 9 = 9$ linearly independent matrices
  - these matrices form a $U(3)$ group
- As was the case for $U(2)$ one matrix is simply the identity matrix multiplied by a complex phase and is of no interest in the context of flavour symmetry
- The remaining 8 matrices have $\det U = 1$ and form an $SU(3)$ group
- The 8 matrices and the Hermitian generators are:

$$\vec{T} = \frac{1}{2}\vec{\lambda}$$

$$\hat{U} = e^{i\vec{\alpha}\cdot\vec{T}}$$

$\boldsymbol{SU(3)}$ flavour: $\boldsymbol{u}$, $\boldsymbol{d}$, and $\boldsymbol{s}$

- In $SU(3)$ flavour, the three quark states are represented by

$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- In $SU(3)$, the uds flavour symmetry contains $SU(2)$ ud flavour symmetry which allows us to write the first three matrices

$$\lambda_1 = \begin{pmatrix} \sigma_1 & \begin{matrix} 0 \\ 0 \end{matrix} \\ 0 \;\; 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} \sigma_2 & \begin{matrix} 0 \\ 0 \end{matrix} \\ 0 \;\; 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} \sigma_3 & \begin{matrix} 0 \\ 0 \end{matrix} \\ 0 \;\; 0 & 0 \end{pmatrix}$$

- For $u \leftrightarrow d$ we have the following matrices

$u \leftrightarrow d$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\boldsymbol{SU(3)}$ flavour: $\boldsymbol{u}$, $\boldsymbol{d}$, and $\boldsymbol{s}$

- The third component of the isospin is now written as

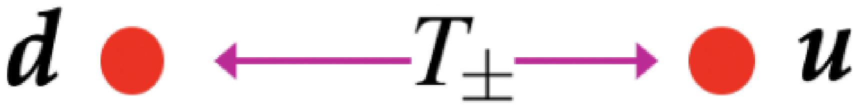
$$I_3 = \frac{1}{2}\lambda_3$$

with

$$I_3 u = +\frac{1}{2}u, \quad I_3 d = -\frac{1}{2}u, \quad I_3 s = 0$$

- I_3 "counts" the number of u quarks minus the number of d quarks in a state
- As before ladder operators

$$T_{\pm} = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$$



$SU(3)$ flavour: u, d, and s

- Now consider the matrices corresponding to the $u \leftrightarrow s$ and $d \leftrightarrow s$

$u \leftrightarrow s$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$d \leftrightarrow s$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Hence in addition to λ_3 there are two other traceless diagonal matrices

- However, the three diagonal matrices are not independent

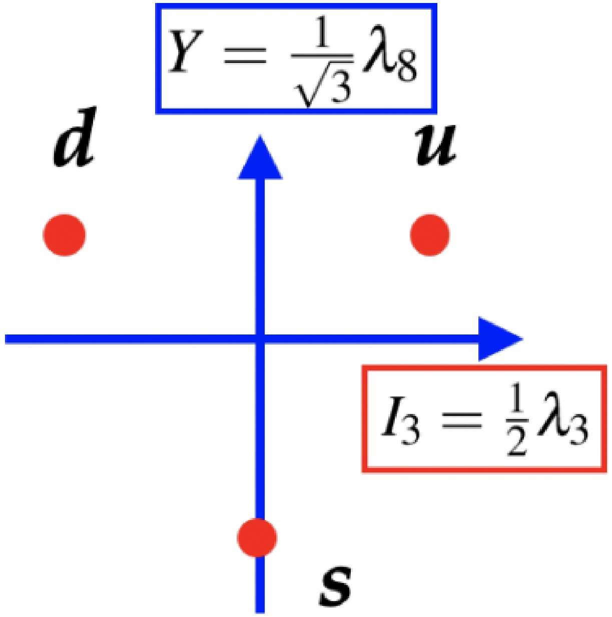
$SU(3)$ flavour: $\boldsymbol{u}$, $\boldsymbol{d}$, and $\boldsymbol{s}$

- Define the eighth matrix, λ_8, as a linear combination

$$\lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

which specifies the "vertical position" in the 2D plane

- We only need two axes (quantum numbers) to specify a state in the 2D plane: (I_3, Y)



$SU(3)$ flavour: u, d, and s

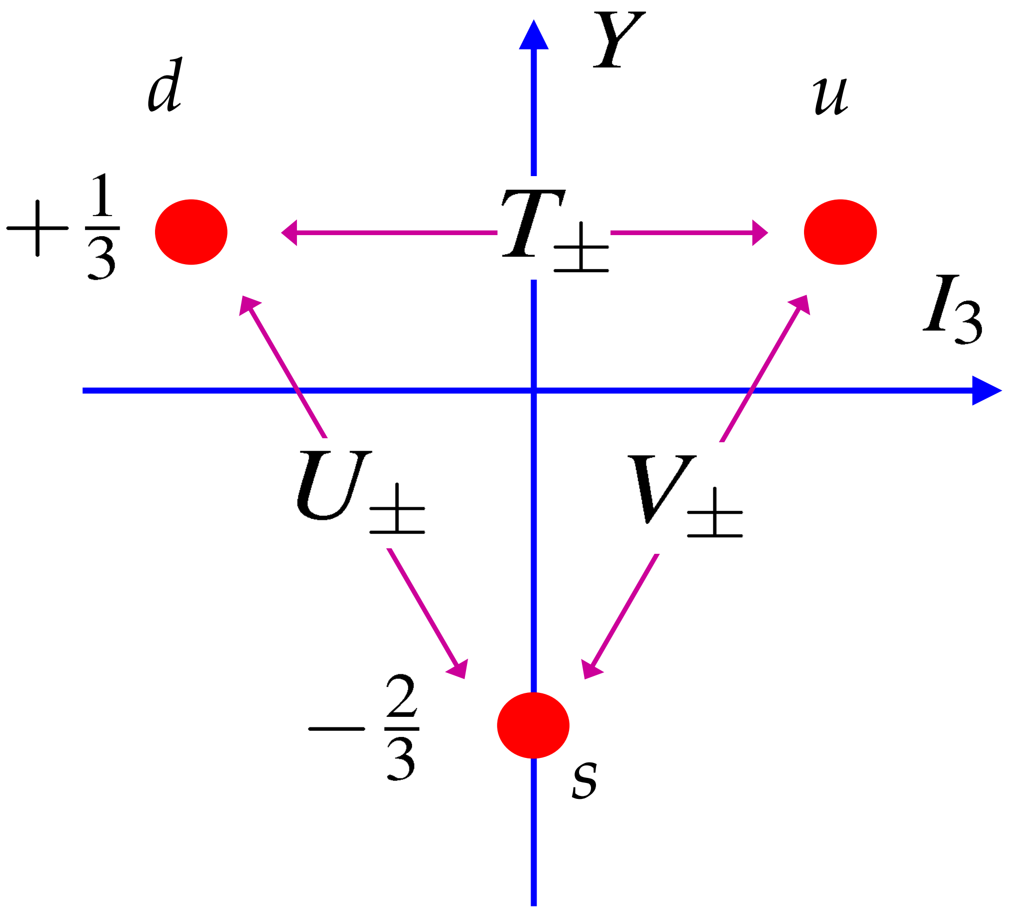
- The other 6 matrices form 6 ladder operators which step between the states

$$T_{\pm} = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$$

$$V_{\pm} = \frac{1}{2}(\lambda_4 \pm i\lambda_5)$$

$$U_{\pm} = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$$

$$I_3 = \frac{1}{2}\lambda_3 \text{ and } Y = \frac{1}{\sqrt{3}}\lambda_8$$



$SU(3)$ flavour: u, d, and s

- The eight **Gell-Mann** matrices

$u \leftrightarrow d$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$u \leftrightarrow s$

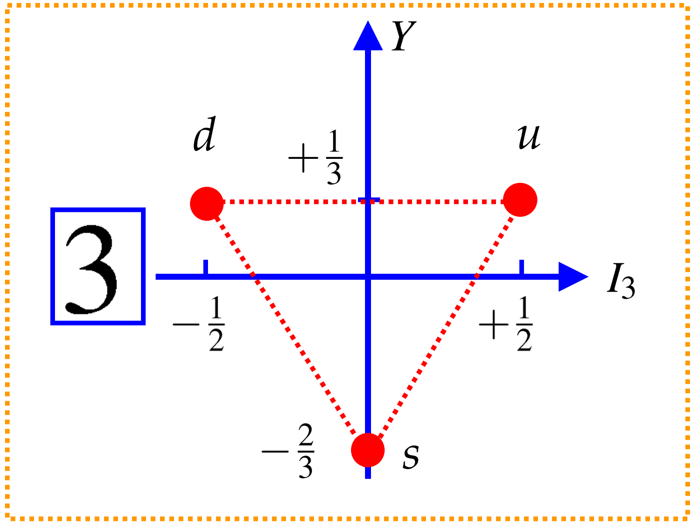
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$d \leftrightarrow s$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Quarks and antiquarks in $\boldsymbol{SU(3)}$ flavour

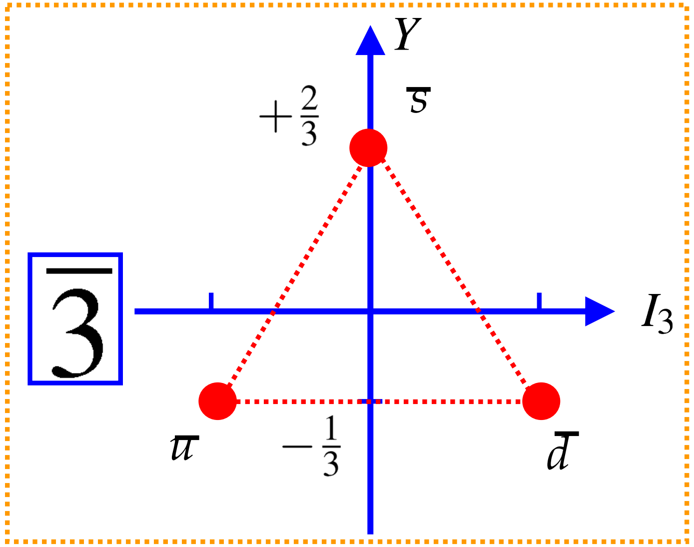


Quarks

$$I_3 u = +\tfrac{1}{2}u; \quad I_3 d = -\tfrac{1}{2}d; \quad I_3 s = 0$$

$$Y u = +\tfrac{1}{3}u; \quad Y d = +\tfrac{1}{3}d; \quad Y s = -\tfrac{2}{3}s$$

- The antiquarks have opposite $SU(3)$ flavour quantum numbers



Antiquarks

$$I_3 \bar{u} = -\tfrac{1}{2}\bar{u}; \quad I_3 \bar{d} = +\tfrac{1}{2}\bar{d}; \quad I_3 \bar{s} = 0$$

$$Y \bar{u} = -\tfrac{1}{3}\bar{u}; \quad Y \bar{d} = -\tfrac{1}{3}\bar{d}; \quad Y \bar{s} = +\tfrac{2}{3}\bar{s}$$

$\boldsymbol{SU(3)}$ ladder operators

- The $SU(3)$ uds flavour symmetry contains ud, us, and ds $SU(2)$ symmetries
- Consider the $u \leftrightarrow s$ symmetry "V-spin", which has the associated $s \to u$ ladder operator

$$V_+ = \tfrac{1}{2}(\lambda_4 + i\lambda_5) = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \frac{i}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

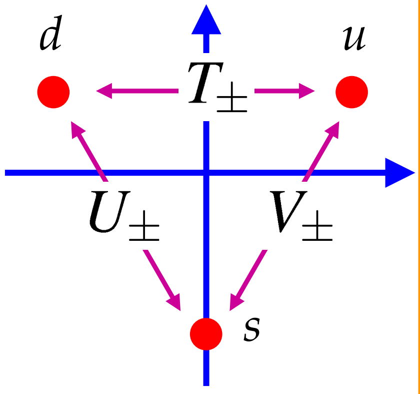
$$V_+ s = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = +u$$

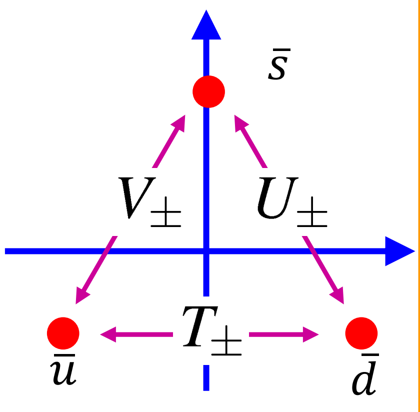
$SU(3)$ ladder operators

$T_\pm = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$

$V_\pm = \frac{1}{2}(\lambda_4 \pm i\lambda_5)$

$U_\pm = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$





$SU(3)$ ladder operators

- The effects of the six ladder operators are

$T_+ d = u; \quad T_- u = d;$	$T_+ \bar{u} = -\bar{d}; \quad T_- \bar{d} = -\bar{u}$
$V_+ s = u; \quad V_- u = s;$	$V_+ \bar{u} = -\bar{s}; \quad V_- \bar{s} = -\bar{u}$
$U_+ s = d; \quad U_- d = s;$	$U_+ \bar{d} = -\bar{s}; \quad U_- \bar{s} = -\bar{d}$

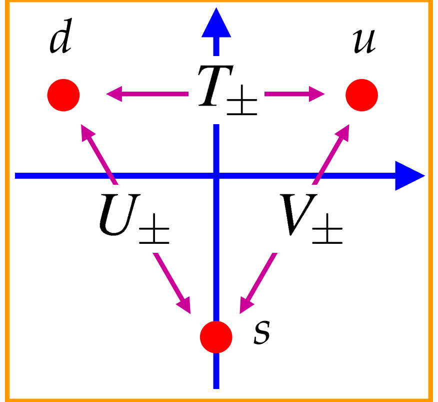
- **All other combinations give 0**

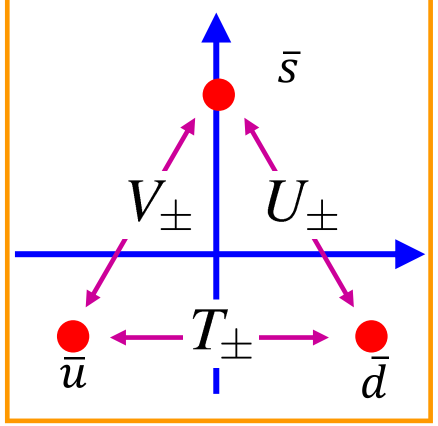
$SU(3)$ ladder operators

$$T_\pm = \tfrac{1}{2}(\lambda_1 \pm i\lambda_2)$$

$$V_\pm = \tfrac{1}{2}(\lambda_4 \pm i\lambda_5)$$

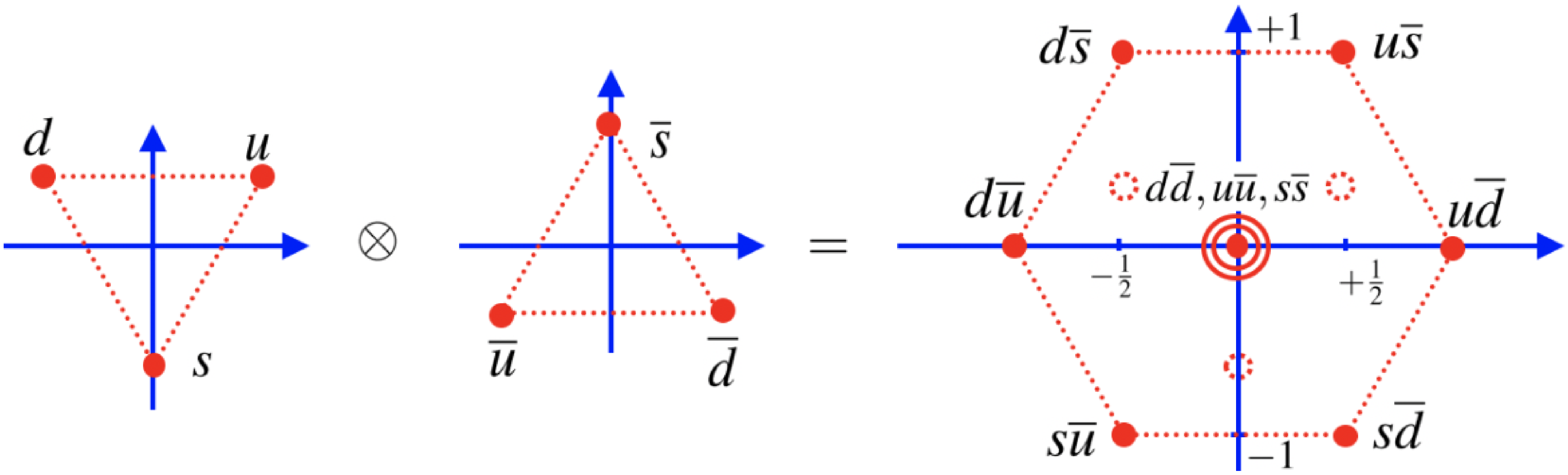
$$U_\pm = \tfrac{1}{2}(\lambda_6 \pm i\lambda_7)$$





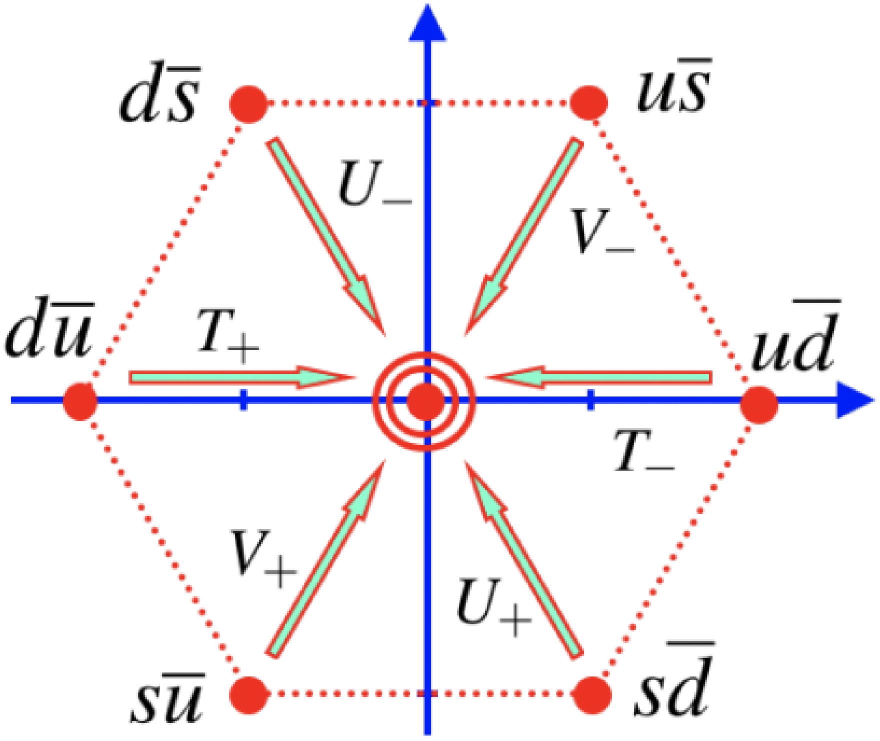
Light $\boldsymbol{uds}$ mesons

- Use the ladder operators to construct uds mesons from the nine possible $q\bar{q}$ states



- The three central states, having $Y = 0$ and $I_3 = 0$ can be obtained using the ladder operators and orthogonality

Light uds mesons



- Starting from the outer states we can reach the center in six ways

$$T_+ |d\bar{u}\rangle = |u\bar{u}\rangle - |d\bar{d}\rangle \qquad T_- |u\bar{d}\rangle = |d\bar{d}\rangle - |u\bar{u}\rangle$$
$$V_+ |s\bar{u}\rangle = |u\bar{u}\rangle - |s\bar{s}\rangle \qquad V_- |u\bar{s}\rangle = |s\bar{s}\rangle - |u\bar{u}\rangle$$
$$U_+ |s\bar{d}\rangle = |d\bar{d}\rangle - |s\bar{s}\rangle \qquad U_- |d\bar{s}\rangle = |s\bar{s}\rangle - |d\bar{d}\rangle$$

- Only two of these six states are linearly independent
- But there are three states with $Y = 0$ and $I_3 = 0$
- One state is not part of the same multiplet $\Longrightarrow$ **cannot be reached with ladder operators**

Light uds mesons

- First form two linearly independent orthogonal states from

$$|u\overline{u}\rangle - |d\overline{d}\rangle \quad |u\overline{u}\rangle - |s\overline{s}\rangle \quad |d\overline{d}\rangle - |s\overline{s}\rangle$$

- If the $SU(3)$ flavour symmetry were exact, the choice of states wouldn`t matter
- However, $m_{\bar{s}} > m_{\bar{u}}, m_{\bar{d}}$ and the symmetry is only approximate
- Experimentally observe three light mesons with $m \sim 140$ MeV: π^+, π^0, π^-
- Identify one state (the π^0) with the isospin triplet (derived previously)

$$\psi_1 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

Light $\boldsymbol{uds}$ mesons

- The second state can be obtained by taking the linear combination of the other two states which is orthogonal to the π^0

$$\psi_2 = \alpha(u\bar{u} - s\bar{s}) + \beta\left(d\bar{d} - s\bar{s}\right)$$

with orthogonality:

$$\langle\psi_1|\psi_2\rangle = 0, \qquad \langle\psi_2|\psi_2\rangle = 1$$

$$\psi_2 = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

- The final state (which is not part of the same multiplet) can be obtained by requiring in to be orthogonal to ψ_1 and ψ_2

$$\psi_3 = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right) - \textbf{singlet}$$

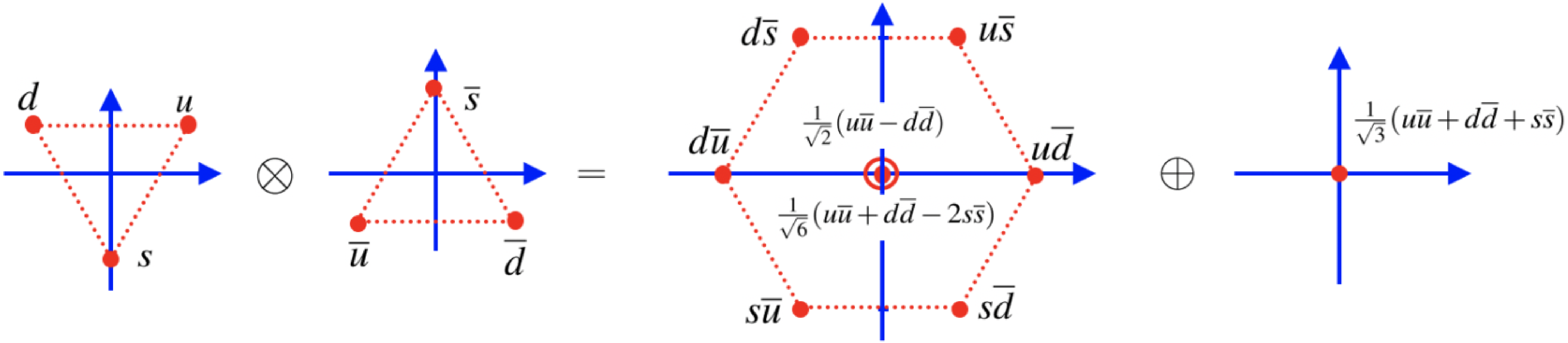
Light uds mesons

- It is easy to check that ψ_3 is a singlet state using the ladder operators

$$T_+\psi_3 = T_-\psi_3 = U_+\psi_3 = U_-\psi_3 = V_+\psi_3 = V_-\psi_3 = 0$$

which confirms that $\psi_3 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} + s\bar{s})$ is a "flavourless" singlet

- Therefore, the combination of quark and antiquark yields nine states which break down into an **octet** and a **singlet**



- In the language of group theory: $3 \otimes \bar{3} = 8 \oplus 1$

Light $\boldsymbol{uds}$ mesons

- Compare with combinations of two spin-half particles $2 \otimes \bar{2} = 3 \oplus 1$

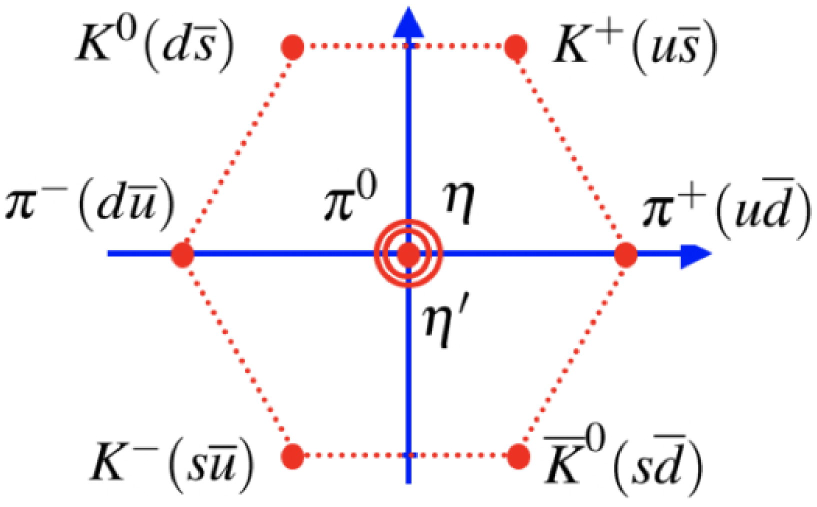
Triplet of spin-one states: $|1,-1\rangle, |1,0\rangle, |1,+1\rangle$

Singlet of spin-one state: $|0,0\rangle$

- These spin triplet states are connected by ladder operators just as the meson uds octet states are connected by $SU(3)$ ladder operators
- The singlet state carries no angular momentum – in this sense the $SU(3)$ flavour singlet is "flavourless"

Pseudoscalar mesons ($L = 0, S = 0, J = 0, P = -1$)

- Because $SU(3)$ flavour is only approximate, the physical states with $I_3 = 0, Y = 0$ can be mixtures of the octet and singlet states
- Empirically we find:



$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$\eta \approx \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

$$\eta' \approx \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \to \text{singlet}$$

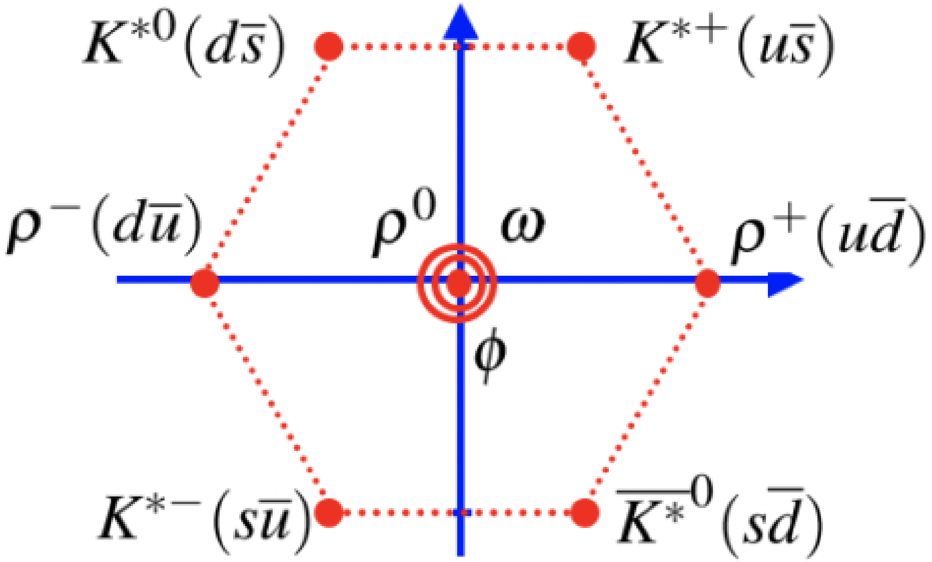
Masses:

$\pi^\pm$: 140 MeV	π^0 : 135 MeV
$K^\pm$: 494 MeV	$K^0/\overline{K}^0$: 498 MeV
η : 549 MeV	η' : 958 MeV

$\rho^\pm$: 770 MeV	ρ^0 : 770 MeV
$K^{*\pm}$: 892 MeV	$K^{*0}/\overline{K^*}^0$: 896 MeV
ω : 782 MeV	ϕ : 1020 MeV

Vector mesons ($L = 0, S = 1, J = 1, P = -1$)

- For the vector mesons the physical states are found to be approximately "ideally mixed"



$$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$\omega \approx \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$\phi \approx s\bar{s}$$

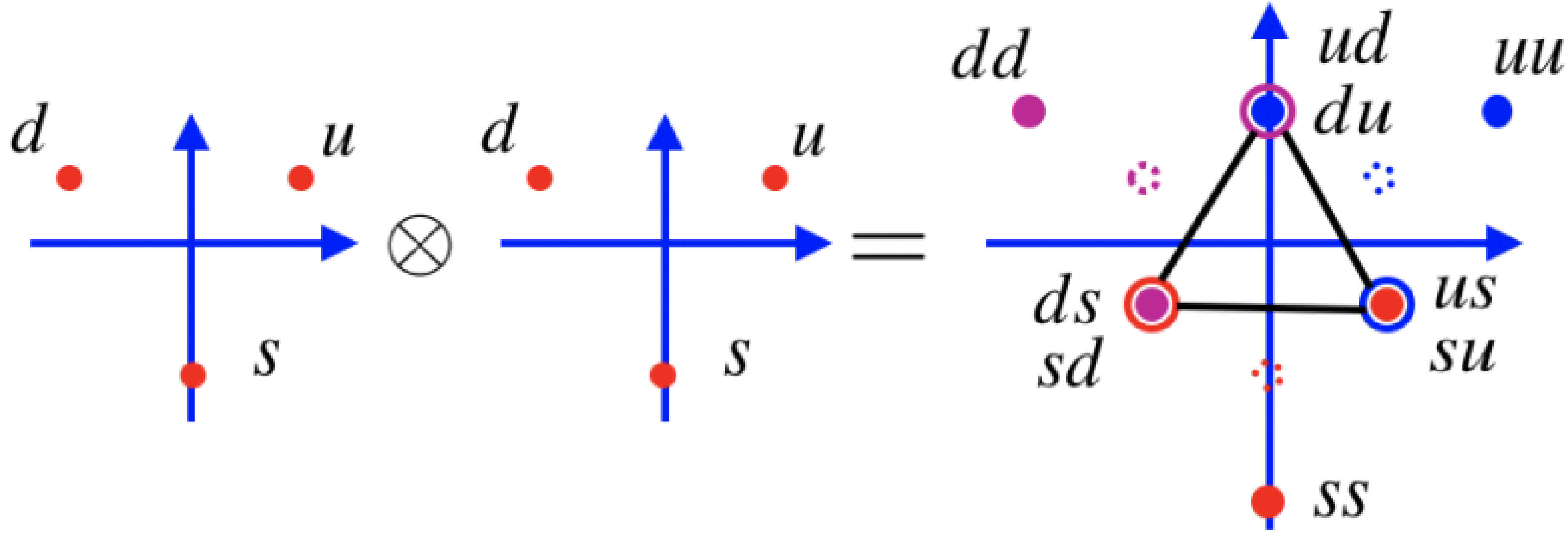
Masses:

$\pi^\pm$: 140 MeV	π^0 : 135 MeV
$K^\pm$: 494 MeV	$K^0/\overline{K}^0$: 498 MeV
η : 549 MeV	η' : 958 MeV

$\rho^\pm$: 770 MeV	ρ^0 : 770 MeV
$K^{*\pm}$: 892 MeV	$K^{*0}/\overline{K^*}^0$: 896 MeV
ω : 782 MeV	ϕ : 1020 MeV

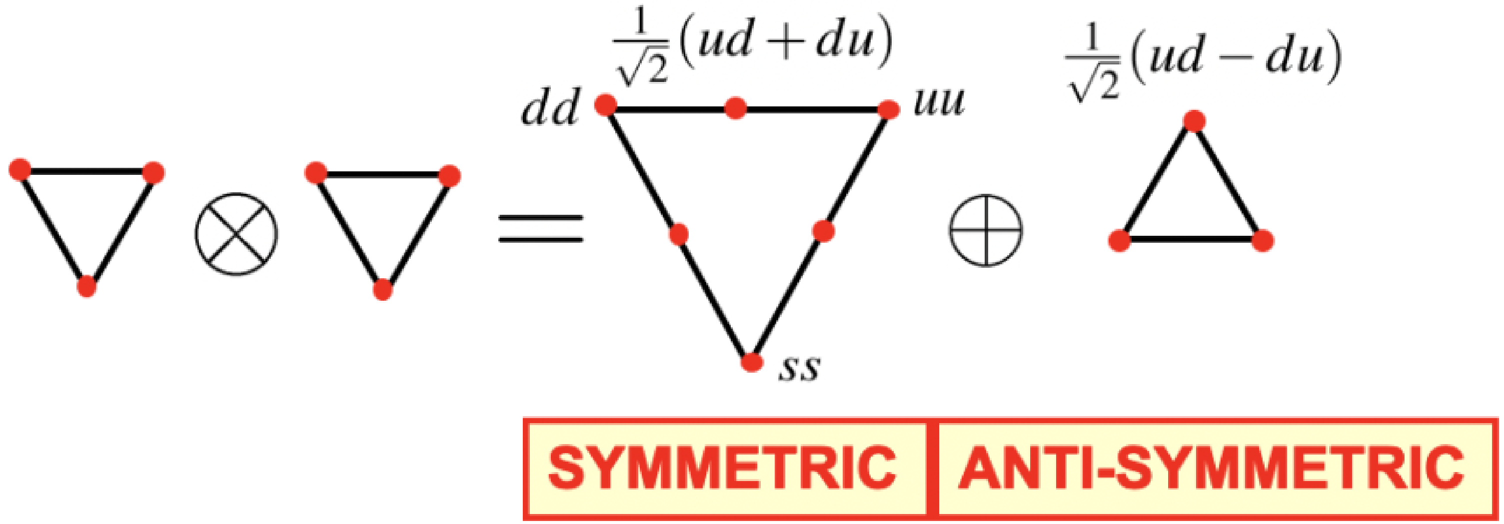
Combining uds quarks to form baryons

- We already saw that constructing baryon states is a fairly tedious process when we derived the proton wavefunction
- Concentrate on multiplet structure rather than deriving all the wavefunctions
- Everything that is done here is relevant to the treatment of color in the future (next semester)
- Once again start by combining two quarks

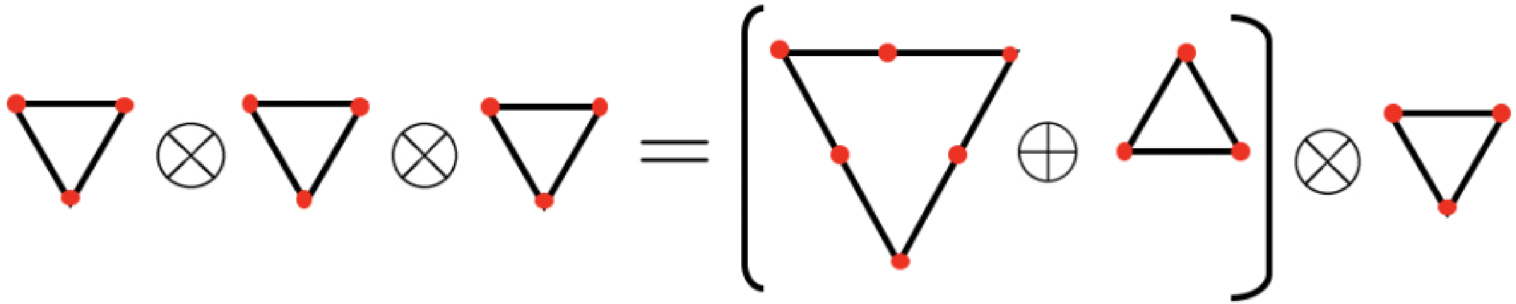


Combining ***uds*** quarks to form baryons

- Yields a symmetric sextet and antisymmetric triplet $3 \otimes 3 = 6 \oplus \bar{3}$: same "pattern" as the antiquark representation



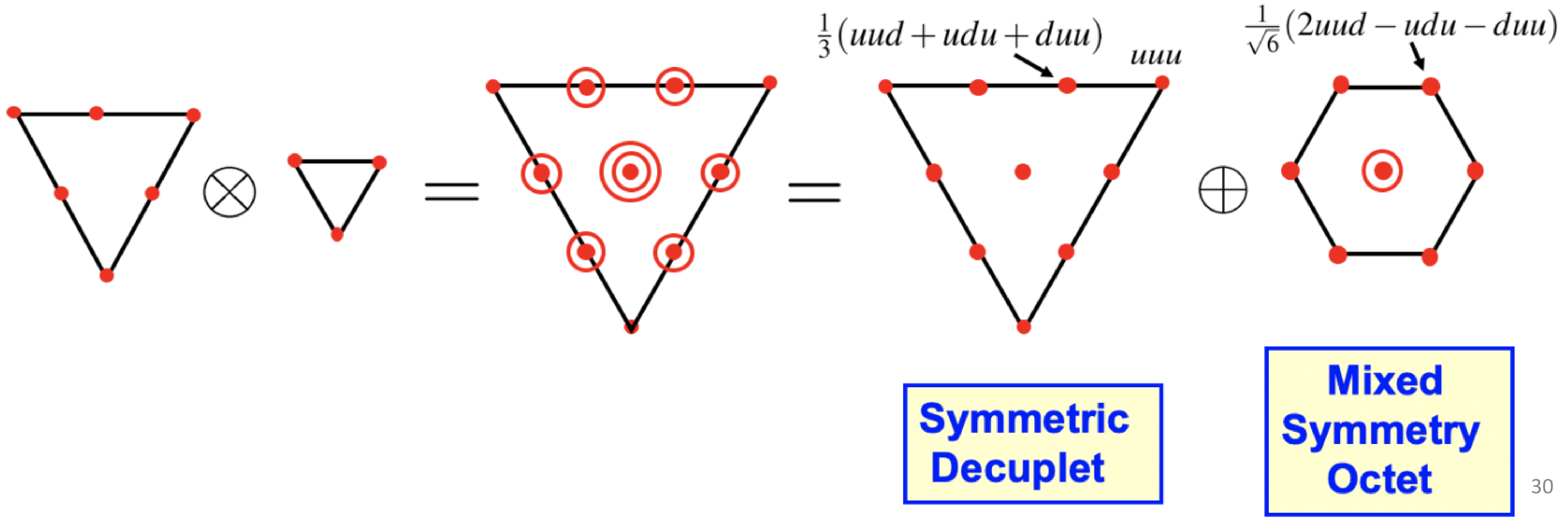
- Now add the third quark



Combining uds quarks to form baryons

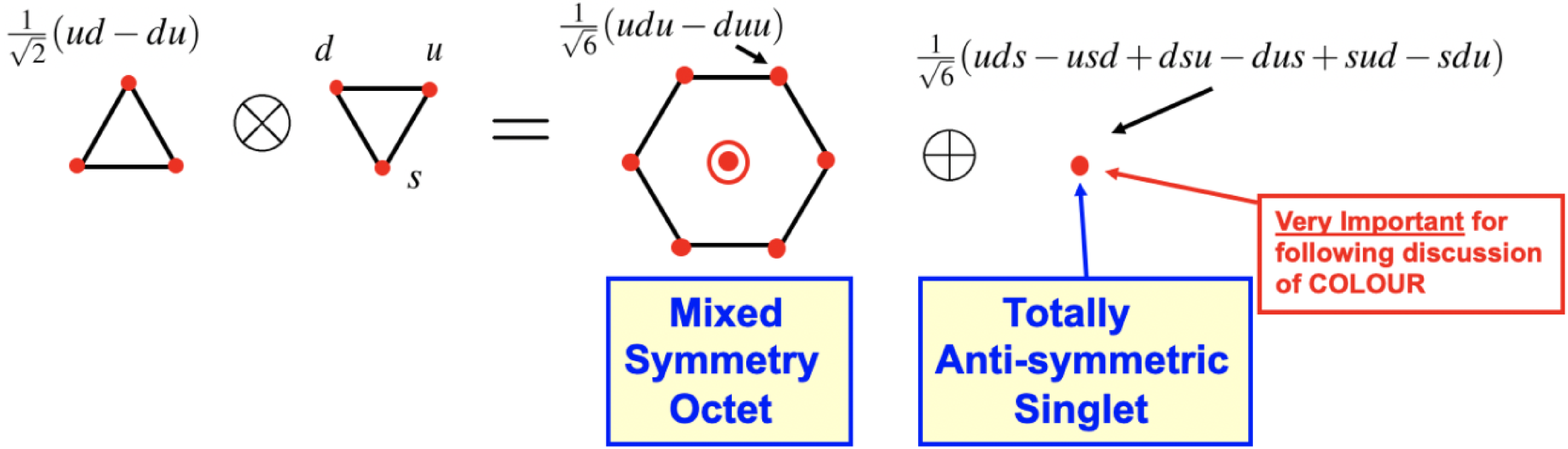
- Best considered in two parts, building on the sextet and triplet
- Again, concentrate on the multiplet structure (for the wavefunction discussions refer to the discussion of proton wavefunction

1. Building on the sextet: $3 \otimes 6 = 10 \oplus 8$



Combining uds quarks to form baryons

2. Building on the triplet: $3 \otimes 6 = 8 \oplus 1$ (same as the case of uds mesons)



Combining $\boldsymbol{uds}$ quarks to form baryons

- We can verify the singlet wavefunction is indeed a singlet by using ladder operators

$$\psi_{\text{singlet}} = \frac{1}{\sqrt{6}}(uds - usd + dsu - dus + sud - sdu)$$

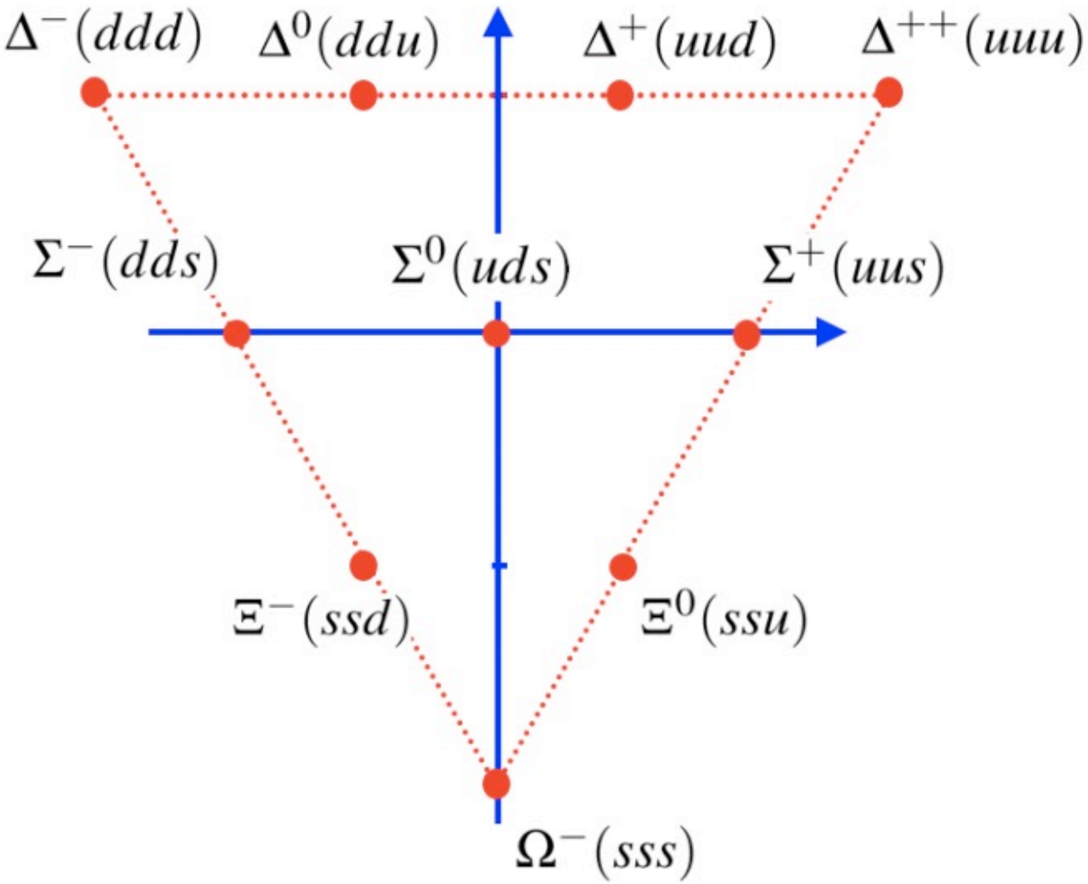
$$T_+\psi_{\text{singlet}} = \frac{1}{\sqrt{6}}(uus - usu + usu - uus + suu - suu) = 0$$

- In summary, the combination of three uds quarks decomposes into:

$$3 \otimes 3 \otimes 3 = 3 \otimes \left(6 \oplus 3\right) = 10 \oplus 8 \oplus 8 \oplus 1$$

Baryon decuplet ($L = 0, S = 3/2, J = 3/2, P = +1$)

- The baryon states ($L = 0$) are the **spin-3/2** decuplet of symmetric flavour and symmetric spin wavefunctions $\phi(S)\chi(S)$



Mass in MeV

Δ(1232)

Σ(1318)

Ξ(1384)

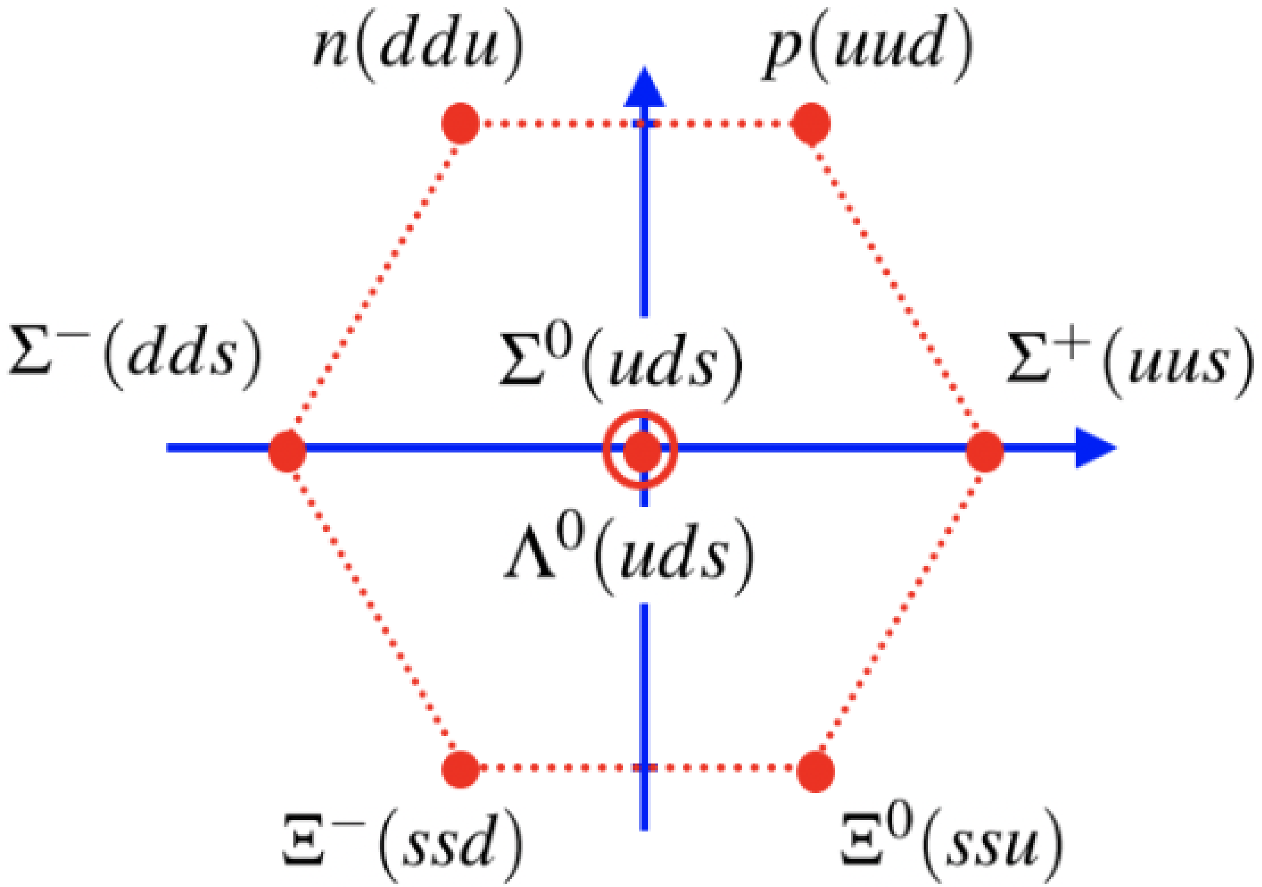
Ω(1672)

Baryon octet ($L = 0, S = 1/2, J = 1/2, P = +1$)

- The **spin-1/2** octet is formed from mixed-symmetry flavour and mixed-symmetry spin wavefunctions

$$\alpha\phi(M_S)\chi(M_S) + \beta\phi(M_A)\chi(M_A)$$

(see previous discussion of proton to see how to obtain wavefunctions)



Mass in MeV

939

Σ(1193)
Λ(1116)

Ξ(1318)

Summary of Lecture 12

Main learning outcomes

- Using the approximate $SU(3)$ symmetry of the Standard Model to construct meson and baryon states
- How to combine quarks (u, d, s) and antiquarks $(\bar{u}, \bar{d}, \bar{s})$ into mesons and baryons
- Classification of hadrons based on their quantum numbers and link to experiment